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INEFFICIENCY IN HOUSE-RENTING MARKETS

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Abstract: Many-to-one market mechanisms are vulnerable to many types of manipulations. This paper investigates the manipulation via capacities in the context of house-renting markets and derives the conditions under which matching mechanisms are immune to manipulation via capacities. We found that the outcome of the matching being Pareto efficient for real estates is a necessary and sufficient condition for non-manipulability via capacities. The main purpose of this paper is to show the relation between the preference ranking and manipulation via capacities.

1. Introduction

The theory of many-to-one matching mechanisms is garnering a great deal of attention, due to its theoretical attractiveness and applicability in real-world problems, such as in the hospital-intern markets and school choice problems in many countries. In a many-to-one matching mechanism, each agent submits their preference ranking and also, agents on one side, their capacities. Throughout the paper, we assume that the preference ranking of the agents is strict, as non-strict preferences can be converted into strict preferences by a tie-breaking lottery, such as, flipping a coin. Beginning with the seminal paper by Gale and Shapley (1962), in which an algorithm was introduced to calculate stable matchings, this field of study has gained a considerable amount of attention from researchers. A matching is stable if no person prefers being unmatched to her assigned house, no real estate agency prefers having a vacant house rather than filling it with one of its matched customers, and there exists no unmatched house-customer pair such that the real estate agency prefers the customer to one of its assignments or keeping a vacant house and the customer prefers the house offered by the agency to her assigned house.

Although many-to-one matching mechanisms are in application in various markets, it is unfortunately fallible to many types of manipulations by the participants in the market. Sonmez (1997) introduced the possibility of another manipulation which is of theoretical and practical interest, the manipulation via capacities. A real estate agency may underreport its capacity to gain better customers. It is further shown that no stable matching mechanism is non-manipulable via this manipulation.

Our paper is closely related to Kesten (2011) who investigates the conditions under which COSM is non-manipulable via capacities. Kesten (2011) assumes that real estate agencies have private information about their preference lists and proposes conditions on entire priority structures. In this paper, we assume that all preference lists are publicly known and we introduce conditions on preference relation of the houses that makes no house better off by manipulating via capacities. We also further generalize the Capacity Lemma of Konishi and Unver (2006) to show that Pareto efficiency for real estate agencies necessarily and sufficiently removes the possibility of manipulation via capacities.

The rest of the paper is organized as follows. Section 2 introduces the preliminary set-up of our model and the existing mechanisms in the literature. In section 3, we provide our results. We show that the matching outcome being Pareto efficient for real estate agencies is necessary and sufficient for non-manipulability via capacities for all matching mechanisms. Section 4 concludes the paper.

2. The model

A house-renting problem is a four-tuple $\Gamma = (H, C, R, q)$. $H \equiv \{h_1, h_2, \dots, h_n\}$, the set of real estate agencies and $C \equiv \{c_1, c_2, \dots, c_n\}$, the set of customers are two non-empty, finite and disjoint sets of agents. $q \equiv (q_h)_{h \in H}$ is a profile of non-negative integers, where q_h is the number of customers that real estate agency *h* is willing to admit. We say that for $A, B \in \{H, C\}$ and $A \neq B$, an element $j \in A$ is acceptable to $i \in B$ if $jR_i\emptyset$ holds. $R \equiv (R_i)_{i \in H \cup C}$ is a list of preference relations of the agents, which are complete and transitive. The corresponding strict preference relations are given as $P \equiv (P_i)_{i \in H \cup C}$. The preference relation R_c of each customer is a binary relation on

$H \cup \{\emptyset\}$ where $\{\emptyset\}$ denotes the case of being unmatched. Since preference relation of customers is complete and transitive, it may be defined as $hP_c h' \Leftrightarrow \forall h, h' \in H, h \neq h'$ and $hR_c h'$. The preference relation R_h of each real estate agency is a binary relation on $2^{|C|}$, which is a set of subsets of customers. Since the preference relation of the real estate agencies is complete and transitive, it may be defined as as $C'P_h C'' \Leftrightarrow \forall C', C'' \in C, C' \neq C''$ and $C'R_c C''$. In addition to the above, we will assume throughout the paper that the preference relation of the real estate agencies is responsive. That is, agencies always prefer a higher ranked customer to a lower ranked customer on its preference order, no matter who the other customers are. As it can be shown, customers' preference relation trivially satisfies responsiveness.

For a given capacity profile q, a matching is a correspondence $\mu: H \cup C \rightarrow 2^{H \cup C}$ such that (i) $\forall c \in C$, $|\mu(c)| \leq 1$ and $|\mu(c)| \subseteq H \cup \emptyset$ (ii) $\forall h \in H$, $|\mu(h)| \leq q_h$ and $|\mu(h)| \subseteq 2^{|C|}$, (iii) $\forall (h, c) \in H$xC, $\mu(c) = h \Leftrightarrow c \in \mu(h)$ [1]. In other words, a matching assigns each customer to at most one house and each real estate agency to at most its reported capacity of customers. Let $M(\Gamma)$ be the set of matchings of Γ. Since matching μ_i is an element of the other set, agent i will have preference ranking on the matching. Agent i prefers matching μ to μ' will be defined as $\mu_i P_i \mu'_i$.

Customer Optimal Stable Mechanism (COSM)

COSM is also known as the Deferred Acceptance algorithm, was devised by Gale and Shapley in the context of college admission. As we will consider COSM in this paper, we will introduce the algorithm.

- Step 1: Every customer applies to her first choice house. Then, if any house's capacity is in excess, the less preferred customers on that agency's list are rejected and the remaining customers under its capacity are tentatively matched with it.

[1] For the sake of concreteness, we will write $\mu(h)$ as μ_h in the paper.

- Step k, $k > 1$: Every customer who was rejected in Step k applies to her kth preferred house. Then, since customers are applying to a house, some agencies when considering the applications it received and the tentative matches it holds, it may be in excess of its capacity. If any house's capacity is in excess, the less preferred customers on the agency's list are rejected and the remaining customers under its capacity are tentatively matched with it.

The algorithm terminates when every customer is matched to a house or every unmatched customer has been rejected by every house on her list of acceptable houses.

3. Manipulation via capacities

Sonmez (1997) proved that there is no stable mechanism that is immune to manipulation via capacities, we will seek the conditions under which a mechanism is non-manipulable via capacities. The mechanisms given above is now a game form where agencies report their capacities as well as their preference ranking and customers report their preference ranking. For each agency $h \in H$ let q_h denote the true capacity of that agency house and let $q_h' \le q_h$ denote the reported capacity of that house. We consider only underreporting of capacities as it is easily shown that overreporting is weakly dominated by truthful capacity revelation. Also, let q_{-h} denote the capacities of houses belonging to other than agencies than $h \in H$. Then we can define the non-manipulability via capacities in the following way:

Definition 1. *A matching mechanism is non-manipulable via capacities if, for all house-renting problems Γ, $\forall h \in H$ and $\forall q_h'$, $\mu_h(\Gamma) R_h \mu_h(H, C, R, q_h', q_{-h})$ holds*[2].

Lemma 1. (Monotonicity) *A house-renting problem is non-manipulable via capacities if and only if , $\forall h \in H$ and $\forall q_h'$, $\mu_h' \subset \mu_h$.*

Proof: non-manipulable via capacities $\Rightarrow$ monotonicity.

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[2] In the same reason to the above, we will write $\mu_h(\Gamma)$ as μ_h and $\mu_h(H, C, R, q_h', q_{-h})$ as μ_h' in the paper.

First of all, let us suppose that the mechanism is non-manipulable via capacities, i.e., $\forall h$, $\forall q_h'$, $\mu_h R_h \mu_h'$. Then suppose $\mu_h' \not\subset \mu_h$, for some agency $h \in H$ and q_h'. Then there exists a set of customers who will be newly matched with agency $h \in H$ and also there must be customers who got rejected to accommodate the new customers, i.e., $\mu_h' \backslash \mu_h \neq \emptyset$ and $\mu_h \backslash \mu_h' \neq \emptyset$. It can be easily found that $|\mu_h'| < |\mu_h|$ holds. Let us use a counterexample. Let there be two agencies, $H = \{h_1, h_2\}$ and three customers, $C = \{c_1, c_2, c_3\}$. Let the preferences of the agencies be: $\{c_1, c_2\} P_{h_1} \{c_1, c_3\} P_{h_1} c_1 P_{h_1} \{c_2, c_3\} P_{h_1} c_2 P_{h_1} c_3$ and $c_3 P_{h_2} c_1 P_{h_2} c_2$. Let the capacities be: $q_{h_1} = 2$, $q_{h_2} = 1$.

Let preferences of the customers be: $h_2 P_{c_1} h_1$ and $h_1 P_{c_2} h_2$.

Then, $\mu_{h_1} = \{c_2, c_3\}$, $\mu_{h_2} = \{c_1\}$. However, if agency h_1 underreport its capacity, the outcome will be $\mu_{h_1}' = \{c_1\}$, $\mu_{h_2}' = \{c_3\}$. Thus, $|\mu_{h_1}'| < |\mu_{h_1}|$ and $\mu_{h_1} P_{h_1} \mu_{h_1}'$, which is a contradiction to the assumption that the mechanism is non-manipulable via capacities.

monotonicity $\Rightarrow$ *non-manipulable via capacities*.

$\mu_h \backslash \mu_h' \neq \emptyset$ and $\mu_h' \backslash \mu_h = \emptyset$. Since every customer $c \in \mu_h$ of every agency $h \in H$ is acceptable for that agency, it follows that $\forall c \in \mu_h \backslash \mu_h'$, $c R_h \emptyset$. Also, the preference ranking of real estate agency, R_h for all $h \in H$ is responsive. Thus, $\mu_h P_h \mu_h'$ and there will be no manipulation via capacities. Q.E.D.

It says that if by manipulating, agencies cannot gain a different customer to those who will be matched to it under truthful capacity revelation, then houses cannot manipulate via capacities. It is a rather strong condition. It is the necessary and sufficient condition for a house admission problem to be non-manipulable via capacities because if the outcome of a matching satisfies the monotonicity condition, it is Pareto efficient for real estate agencies.

Proposition 1. *Given a preference ranking of the agencies, a matching mechanism achieves Pareto efficiency for agencies if and only if monotonicity holds.*

Proof: Pareto efficiency $\Rightarrow$ *monotonicity.*

Suppose that the outcome of matching is Pareto efficient for the agencies, i.e., there is no matching μ' that Pareto dominates it.

Then by Lemma 1, the outcome of matching μ can be Pareto inefficient, which is a contradiction.

monotonicity ⇒ Pareto efficiency.

Suppose that monotonicity holds. Then suppose that the outcome of matching μ is not Pareto efficient for agencies. Then there exists another matching μ' that Pareto dominates it. Let $C' = \{h \in H \mid \mu'_h P_h \mu_h\}$. If there exists another matching that Pareto dominates μ then this set is non-empty, i.e., $C' \neq \emptyset$. Let $S' = \mu'_h \backslash \mu_h$ for some $h \in C'$. $S' \neq \emptyset$, because if $S' = \emptyset$, then it cannot be $\mu'_h P_h \mu_h$ for some $h \in H$. Thus $\mu'_h \backslash \mu_h \neq \emptyset$. It implies $\mu'_h \not\subset \mu_h$, which is a contradiction. Q.E.D

4. Conclusion

Our results show that it is possible for the Deferred Acceptance algorithm to be immune to manipulation via capacities. We found that Pareto efficient outcome for real estate agencies is the necessary and sufficient condition for two-sided matching markets to be non-manipulable via capacities. We aimed to find the relation between preference of the agencies and manipulation via capacities and the motivation of such strategic behavior on the part of the houses in this paper. One direct implication of our results concerning Pareto efficiency for real estate agencies is that it bodes bad news for customers because since two-sided matching is a lattice, the matching whose outcome is Pareto efficient for houses is the worst possible matching for the customers.

5. References

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